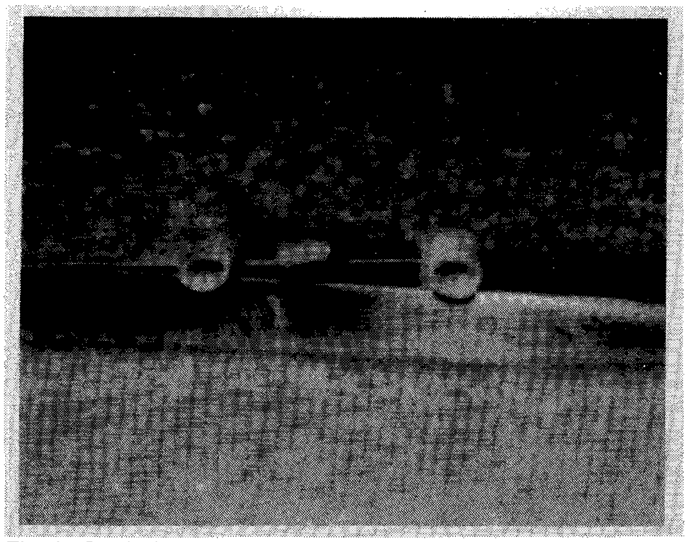

Fig. 6. Luminal view of rabbit artery before (center) and after (sides) *in vitro* balloon microwave angioplasty.

IV. Conclusion

Preliminary studies utilizing microwave balloon angioplasty at 2450 MHz were conducted. Local temperatures were raised to 90°C with a power of 15 W into the 0.032-in.-diameter coaxial cable, producing localized tissue modifications in both dog myocardium *in vitro* and rabbit arteries *in vivo*.

Acknowledgment

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A Rigorous Variational Formulation of an *H* Plane Slot-Coupled Tee Junction

B. N. DAS, N. V. S. NARASIMHA SARMA, AND A. CHAKRABORTY

***Abstract* — The paper presents a rigorous analysis for the determination of an exact equivalent network for the slot-coupled *H* plane Tee junction, taking the effect of wall thickness into account. A variational formulation is used for the determination of the parameters of the network. The parameters of the *S* matrix as well as coupling are evaluated from the exact equivalent network. Comparisons between theoretical and experimental results wherever available are presented.**

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The authors are with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur - 721 302 India.
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I. Introduction

In the investigations of slot-coupled Tee junctions reported in the literature [1] using the variational method, no rigorous analysis of the effect of wall thickness has been presented and no attempt has been made to evaluate the S matrix of the junction.

In the present work, a variational formulation similar to that used by Oliner [2] for a radiating slot is used for the evaluation of network parameters of a slot-coupled Tee junction taking the effect of wall thickness into account. This formulation demands a knowledge of dyadic Green's functions in the primary and coupled guides, which are evaluated using the formulation available in the literature [3]–[5].

The coupling slot in a waveguide wall of finite thickness is represented as a section of transmission line. In the equivalent network, this transmission line is coupled to the primary and secondary guides by transformers whose turns ratios are also evaluated.

The equivalent network of the three-port junction is used for the evaluation of S parameters as well as coupling between the primary and secondary guides. A comparison between theoretical and experimental results on coupling is presented.

II. Analysis

Fig. 1(a) shows the slot-coupled Tee junction and the coordinate system. The equivalent circuit of the junction is presented in Fig. 2.

At the cross section $Y' = 0$ plane of the guide, the electric field is of the form

$$E_{\text{slot}} = \begin{cases} E_1 \cos\dfrac{\pi z'}{2L} & \begin{matrix} -L < z' < L \\ -W < x' < W \end{matrix} \\ 0 & \text{elsewhere.} \end{cases} \tag{1}$$

This electric field in the cross section is replaced by equivalent magnetic current. The admittance presented by the Tee arm at the secondary terminals of T_2 is obtained from the self reaction of this current with the magnetic field in the coupled guide.

Expressing the magnetic field in the coupled guide in terms of modal voltages V^e_{mn} and V^m_{mn} of the TE and TM modes, the admittance presented at the interface D is obtained as

$$Y_{2s} = \frac{\sum\limits_m \sum\limits_n \left[V^{e^2}_{mn} y^e_0 + V^{m^2}_{mn} y^m_0 \right]}{V^e_{10} \cdot V^e_{10}} \tag{2}$$

where V^e_{10}, the dominant mode modal voltage in the secondary guide, is given by

$$V^e_{10} = \oint E_{\text{slot}} \cdot e^e_{10}\, ds = \sqrt{\frac{2}{ab}}\, \frac{4W \dfrac{\pi}{2L} \cos\dfrac{\pi L}{b}}{\left(\dfrac{\pi}{2L}\right)^2 - \left(\dfrac{\pi}{b}\right)^2} E_1. \tag{3}$$

Y^e_0 and Y^m_0 are the wave admittances of TE and TM modes respectively and e^e_{10} is the dominant mode vector function.

The admittance presented at the primary of transformer T_2 is given by

$$Y_2 = Y_{2s} n_2^2 \tag{4}$$

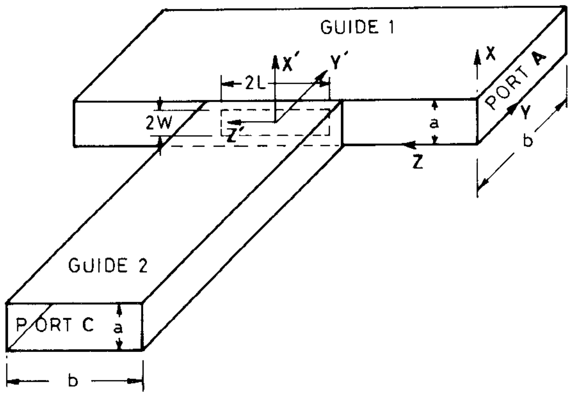

Fig 1 Slot-coupled H plane Tee junction.

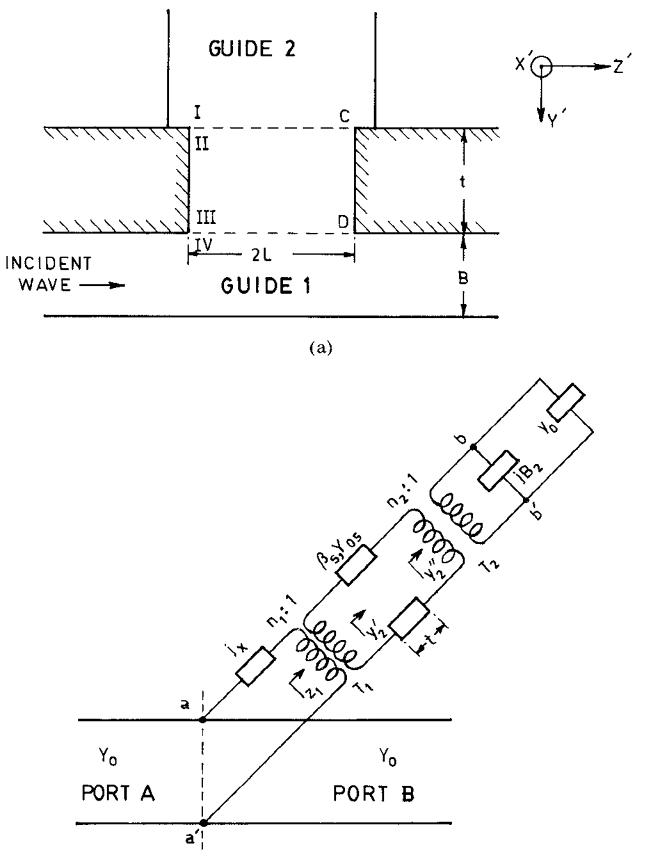

Fig 2. (a) Illustrating slot as stub waveguide. (b) Equivalent circuit representation of the junction.

where [5]

$$\frac{1}{n^2}=\sqrt{\frac{V_{\text{slot}}V_{\text{slot}}}{V_{10}^e V_{10}^e}}. \tag{5}$$

Hence V_{slot} is the dominant mode modal voltage in the stub waveguide. Using the formulation suggested by Harrington [5], V_{slot} is obtained as

$$V_{\text{slot}}=\sqrt{2LW}\,E_1.$$

This admittance Y_2 terminates a transmission line with wave admittance Y_{0s} and propagation constant γ_s. The admittance which appears across the secondary of transformer T_1 is found by using [6, eq. (3.91)]. The constant γ_s is real for $\lambda < 4L$ and is imaginary for $\lambda > 4L$.

At interface C, the reaction on both sides of the slot interface is conserved. Use of this property leads to the following relation with the aid of equations in [5, pp. 118–119, 421]:

$$V_{\text{slot}}^{10}\cdot V_{\text{slot}}^{10}\cdot Y_2' = -I\cdot I\cdot Z_1 \tag{6}$$

where I is the discontinuity in modal current, which is the same as the current flowing into the primary of transformer T_1. The expression is given by [2, eq. (8)] and is reproduced below:

$$I = jY_0\oint \hat{n}\times E_{\text{slot}}\cdot r^{(2)}\,ds$$

$$r^{(2)} = j\sqrt{\frac{2}{ab}}\,\sin\frac{\pi y}{b}\,\sin(\beta_{01}Z)\hat{U}_x + j\sqrt{\frac{2}{ab}}\,\frac{\pi}{b\beta_{01}}\cos(\beta_{01}Z)\hat{U}_Z.$$

Z_1, the impedance presented at the primary of the transformer, is given by

$$Z_1=\frac{n_1^2}{Y_2'}. \tag{7}$$

The transformer turns ratio is defined as

$$\frac{1}{n_1}=\sqrt{\frac{\left(\oint \hat{n}\times E\cdot r^{(2)}\,ds\right)^2}{V_{\text{slot}}V_{\text{slot}}Y_2'^2}}. \tag{8}$$

The circuit parameter jX appearing in Fig. (2) is obtained by using the following expression [2]:

$$jX=\frac{\iiiint_{\text{slot}} \hat{n}\times E(x,z)\cdot j\bar{\bar{B}}\cdot \hat{n}\times E(x',z')\,dx'dz'dx\,dz}{Y_0^2\left[\iint \hat{n}\times E\cdot r^{(2)}\,dx\,dz\right]^2}. \tag{9}$$

The quantity $j\bar{\bar{B}}$ is given by the expression [7]

$$j\bar{\bar{B}}=\frac{j}{\omega\mu}\left(K^2\bar{\bar{I}}+\nabla\nabla\right)G(r,r') \tag{10}$$

where $\bar{\bar{I}}$ is unit dyadic and K is the wavenumber.

Using the expression for $G(r,r')$ of [4] and [5], the expression for jX is obtained as

$$jX=\frac{\dfrac{j}{\omega\mu}\displaystyle\sum_m\sum_n\frac{\epsilon_m\epsilon_n}{2\gamma ab}\left(\frac{2W\cos\frac{n\pi}{2}\,\sin\frac{n\pi w}{a}}{n\pi w/a}\right)^2}{-Y_0^2\dfrac{2}{ab}\left(\dfrac{\pi}{b\beta_{01}}2W\dfrac{(\pi/L)}{\left(\frac{\pi}{2L}\right)^2-\beta_{01}^2}\cos\beta_{01}L\right)^2}\cdot\left[\frac{(k^2+\gamma^2)e^{-\gamma L}\left(\frac{2\pi}{2L}\right)^2\cosh\gamma L}{\left\{\gamma^2+(\pi/2L)^2\right\}^2}+\frac{\left(k^2-(\pi/2L)^2\right)2\gamma L}{\gamma^2+(\pi/2L)^2}\right] \tag{11}$$

III. S Parameters and Coupling

From this equivalent network, it is possible to determine the impedance seen by the generator connected to port A when ports B and C are match terminated.

From Fig. 2, the normalized impedance loading between terminals a and a' is given by

$$Z_T = jX + Z_1 = 1/Y. \tag{12}$$

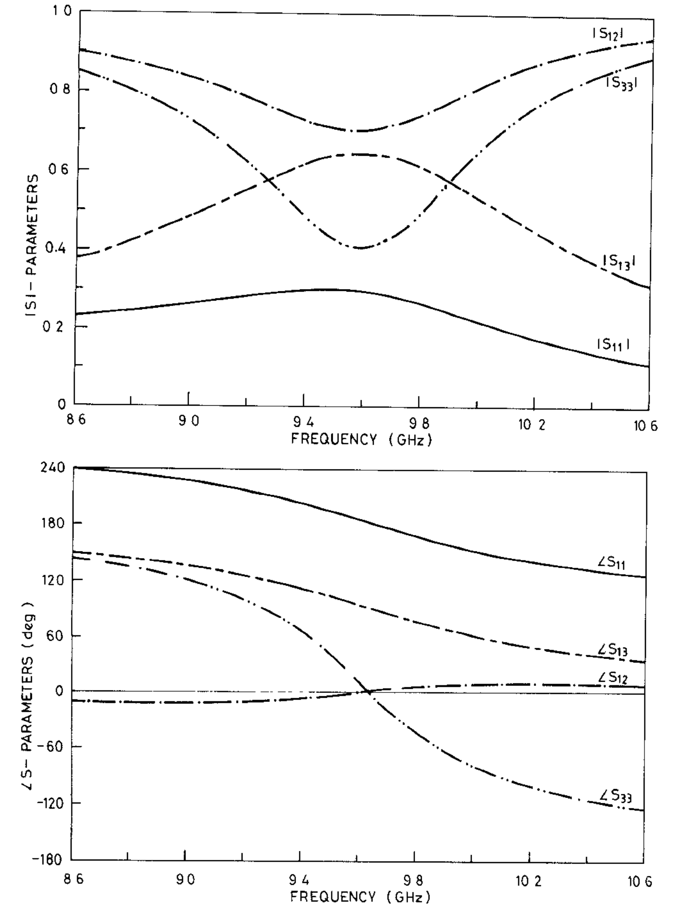

Fig. 3. Variation of S parameters of the junction with frequency for $2L = 16$ mm, $2W = 0.8$ mm, and $t = 1.27$ mm

From this, the parameters of the S matrix are obtained as

$$S_{11} = \frac{-Y}{Y+2} \tag{13}$$

$$S_{12} = 1 + S_{11}. \tag{14}$$

Reciprocity and symmetry conditions lead to the following relations between the elements of the S matrix [8]:

$$S_{21} = S_{12} \qquad S_{22} = S_{11} \qquad S_{31} = S_{13}.$$

To determine the S_{33} parameter, it is necessary to determine the impedance seen by terminals b − b′, which is given by

$$Y_s = \frac{Y'_{0s}}{n_2^2} \frac{\left(\frac{jX + Z_0/2}{n_1^2}\right) + Y'_{0s} \tanh \gamma_s t}{Y'_{0s} + \left(\frac{jX + Z_0/2}{n_1^2}\right) \tanh \gamma_s t} + jB'_{2s} \tag{15}$$

where jB'_{2s} is the imaginary part of Y'_{2s}.

If Y_s is the input admittance seen from port C, then element S_{33} of the S matrix is obtained as

$$S_{33} = \frac{Y_0 - Y_s}{Y_s + Y_0}. \tag{16}$$

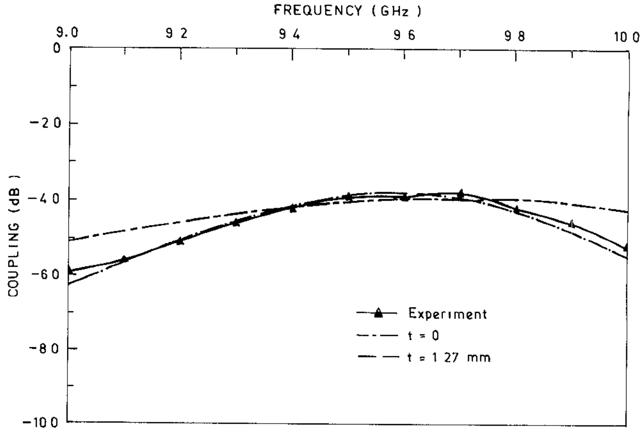

Fig. 4. Variation of coupling (dB) with frequency for $2L = 16$ mm, $2W = 0.8$ mm, and $t = 1.27$ mm, as well as for $t = 0$.

The S_{31} $(= S_{13})$ parameter is determined from the equations which satisfy the unitary property given by

$$S^{\dagger} S = U$$

where $S^{\dagger}$ is complex conjugate of S in view of the symmetry of the S matrix and U is the unit matrix.

An expression for coupling from port A to port C is obtained as

$$\text{coupling (dB)} = 20 \log |S_{31}|. \tag{17}$$

IV. Numerical and Experimental Results

Using (3), (5)–(8), (11), and (13)–(16), the variation of the parameters S_{11}, S_{12}, S_{33}, and S_{13} with frequency has been evaluated for $2L = 16$ mm, $2W = 0.8$ mm, $a = 10.16$ mm, $b = 22.86$ mm, and $t = 1.27$ mm over the range 8.6–10.6 GHz. The results are presented in Fig. 3. The variation of coupling (dB) with frequency for $t = 1.27$ mm and $t = 0$ has been evaluated using (17) and is presented in Fig. 4.

The experimental data on coupling for $t = 1.27$ mm, measured using HP instruments over the frequency 9–10 GHz, are also presented in Fig. 4 for the sake of comparison.

V. Discussion

The analysis presented above permits the evaluation of the exact equivalent circuit of the slot-coupled Tee junction using a variational method with the inclusion of the effect of wall thickness. From this equivalent circuit all the complex elements of the S matrix have been determined. A complete knowledge of the S matrix parameters allows estimation of the electrical performance of the junction for any termination of the ports.

The excellent agreement between the theoretical and experimental results on coupling presented in Fig. 4 justifies the validity of the rigorous analysis presented in the paper.

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